Assessment of Geoidal Undulation (N) Values Computed with Reference to Different Earth Gravity Models

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Abstract: *This research entails the determination of the variations in global geoid undulation with reference to different Earth Gravity Models (EGMs). Geoidal undulation values of 22 points selected within the study area were determined from GPS derived data on differential mode. The undulation values were computed with reference to three different earth gravity models (EGM84, EGM96 and EGM2008) which use geopotential coefficients and spherical harmonic analysis. The results obtained were plotted and the configuration of the local geoid was obtained with respect to each gravity model. It was observed that structural development in the school (study area) since its inception in 1981 has forced some significant changes in the mass distribution in the school and consequently changes in gravity values which in turn changes the undulation values (which depends on gravity) from point to point. It was also observed that there was correlation between the local geoid determined and the nature of the topography. Areas with higher ellipsoidal or orthometric heights experience greater separation between the geoid and the ellipsoid.*
Key Words: Geoid, Geoid Undulation, EGM, DGPS, Topography

I. Introduction

Being the physical surface of the earth, Geoid is the border between the solid or fluid masses and the atmosphere. The ocean floor is included in this formulation, being the bounding surface between the solid terrestrial body and the oceanic water masses. The irregular surface of the solid earth (continental and ocean floor topography) cannot be represented by a simple mathematical (analytical) function. It is therefore described point wise by the coordinates of control points. Given an adequately dense control network, the detailed structure of this surface can be determined by interpolation of data from terrestrial topographic and hydrographic surveying. On the other hand, the ocean surfaces (70% of the earth's surface) are easier to represent. If we neglect the effects of ocean currents and other "disturbances", the ocean surfaces form a part of a level or equipotential surface of the earth's gravity field (surface of constant gravity potential). We may think of this surface as being extended under the continents and identify it as the mathematical figure of the earth, which can be described by a condition of equilibrium (Helmert, 1884). J. B. Listing (1873) designated this level surface as geoid.

The body of the earth and its gravity field are subject to temporal variations of secular, periodic, and abrupt nature, which can occur globally, regionally, and locally. These variations also influence the orientation of the earth. Modern geodetic measurement and evaluation techniques are used to detect these variations to a high level of accuracy. If time-independent results are required, geodetic observations must be corrected for temporal variations. By determining temporal variations, the science of geodesy contributes to the investigation of the kinematic and dynamic properties of the terrestrial body. Accordingly, the figure of the earth and the external gravity field must be considered as time dependent quantities: "Four-dimensional geodesy".

There are relationships between the topographical surface of the earth and its figures of approximations (Geoid and Ellipsoid). These relationships can be established by their height systems. The height of a point on the earth's surface measured along the ellipsoidal normal to the surface of the Ellipsoid is known as ellipsoidal height (h). The height of a point on the earth's surface measured along the plumbline, normal to the Geoid, to the surface of the Geoid is known as orthometric height (H). Orthometric height coincides with the direction of gravity vector which is at all points normal to the surface of the Geoid. Orthometric height at every point therefore is a function of gravity at that point. The surface of the geoid is higher than the reference ellipsoid wherever there is a positive gravity anomaly and lower than the reference ellipsoid wherever there is a negative gravity anomaly. The difference between these two height systems (ellipsoidal and orthometric heights) is the Geoidal Undulation variations (See Fig. 1) that this work sought to determine in the study area. The Geoid Undulation determination in the study area is done by Global Positioning System (GPS) observations on differential mode. The geographical coordinates (Longitude, λ and Latitude, ϕ) and the ellipsoidal height, h has

been obtained from DGPS observations and is used as data from which the Geoid Undulation and Orthometric height were determined. The study of the height relationships between these surfaces with reerence to various Earth Gravity Models (EGMs) will definitely lead to the determination of series of Geoidal undulations. Therefore, undulation of the Geoid is the mathematical process of determining the height in meters above the Geoid (relative to the Mean Sea Level) from height provided by the Global Positioning System (GPS) which uses the World Geodetic Survey (WGS84) ellipsoid as reference. In maps and common use the height over the Mean Sea Level is used to indicate the height of elevations while the ellipsoidal height results from the restrictions that apply for the GPS system. The process of undulation determination is not standardized, as different countries use different Mean Sea Level as reference but this study is referred to (WGS84) ellipsoid.

Separation between the geocentric reference ellipsoid and the geoid is called the geoidal height, or geoidal undulation, and is generally denoted by N. Torge, (2001) stated that if a point is located on the geoid, we obtain the geoid height, N (also called geoidal undulation) as the vertical distance between the ellipsoid and the geoid. A geometric definition follows by differencing the ellipsoidal height, h and the orthometric height, H as represented by equation (1):

$$N = h - H \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1)$$

Where N = Geoidal Undulation, h = Ellipsoidal height and H = Orthometric height
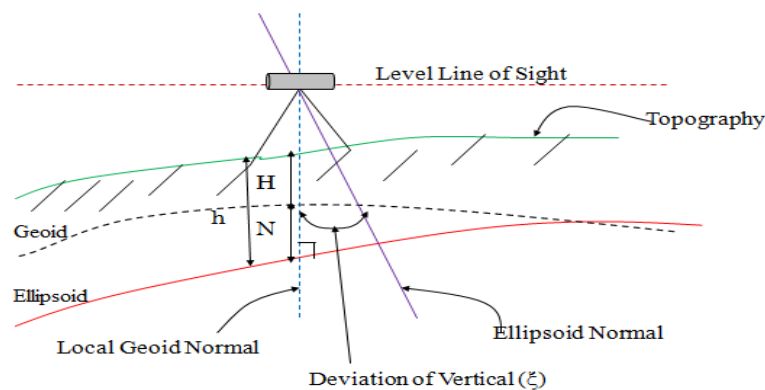

Figure 1: Relationship between Ellipsoidal and Orthometric Heights.

In recent times, the advent of the Global Positioning System (GPS) has helped tremendously in reducing the labour involved as well as in enhancing the accuracy of positioning. The GPS constitutes best known satellite navigation system that provides autonomous geo-spatial positioning with global coverage. The satellites orbiting about the centre of gravity of the earth, can only measure heights relative to a geocentric reference ellipsoid. To obtain one's geoidal undulation as to determine the geoidal height, a raw GPS reading must be converted by the use of Geoid Height Calculator/ EGM96 geoid calculator/GeoidEval Software. In this study, global geoidal undulations variations in Modibbo Adama University of Technology Yola, Adamawa State were determined through the determination of ellipsoidal coordinates of selected points using GPS observations on differential mode, determining the Geoidal Undulation and orthometric heights using Geoid Height Calculator, EGM96 Geoid Calculator and GeoidEval Calculator Software and assessing the geoidal undulation (N) values computed with reference to different Earth Gravity Models (EGMs) over the area of survey.

II. Study Area
The study area covers the entire campus of Modibbo Adama University of Technology Yola, Gerei Local Government Area, Adamawa State, Nigeria. It is between longitude 12° 29'E and 12° 31'E; latitude 9° 19'N and 9° 22'N (See Fig. 2). Some parts of the school are rocky with most of the ground covered with sandy soil of relatively flat terrain. The area studied was approximately 7.98Km^2.
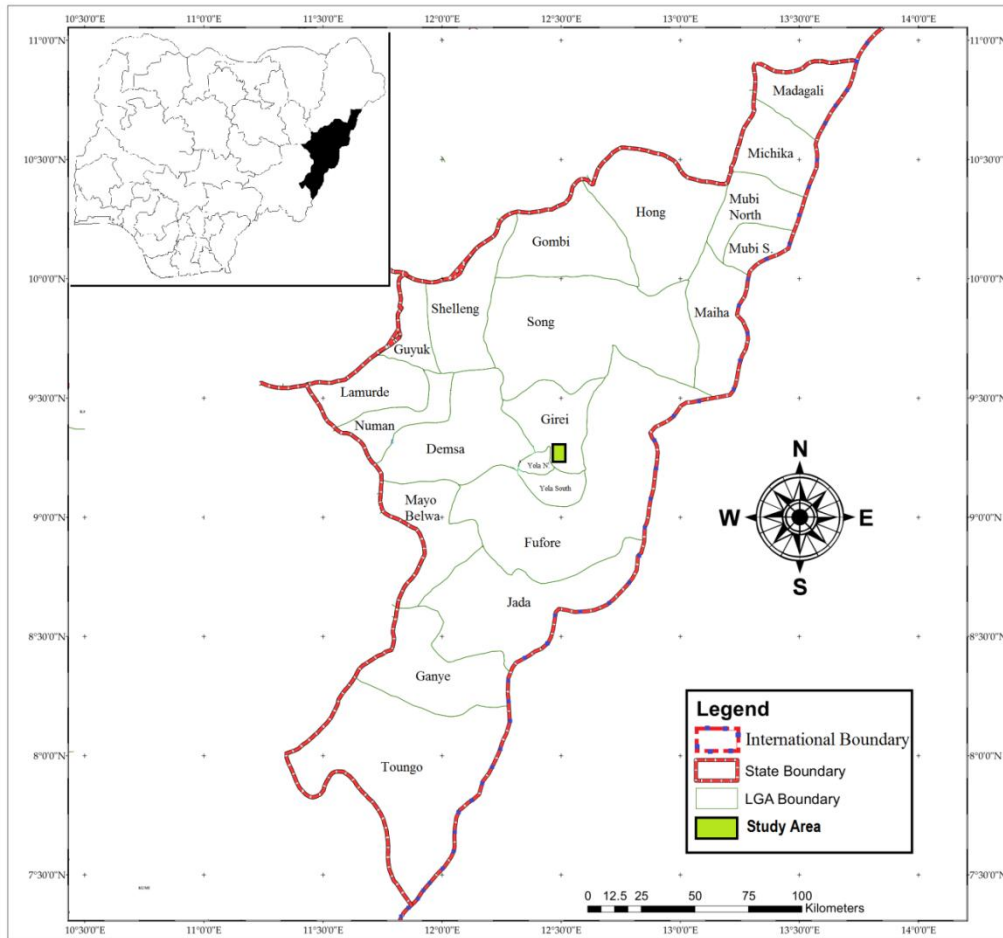
Figure 2: Adamawa State Showing the Study Area
Source: Space Applications Department, NASRDA (2006)

III. Materials and Methods

Fig. 3 is the Methodology Workflow that shows step by step procedure used in the execution of this study.
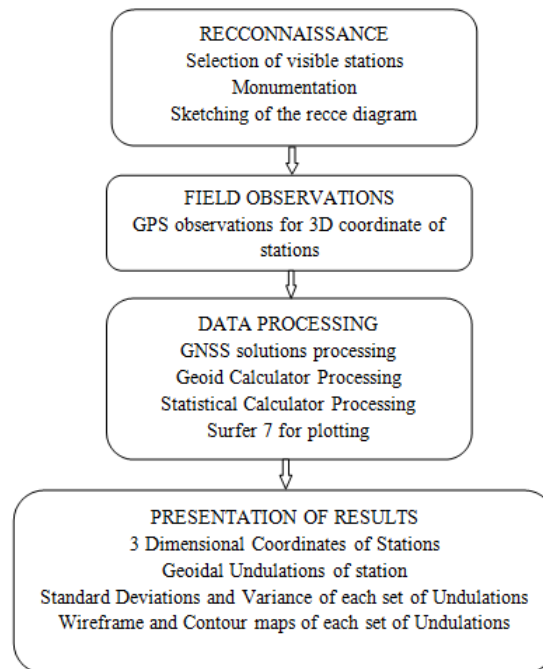
Figure 3: Methodology Workflow

3.1 Data Acquisition

One of the fundamental steps to successful research work is data acquisition method related to the research. The output required depends on the input data acquired. Global Positioning System on Differential mode was used to acquire field data for this research. Twenty-two stations were marked in the study area during reconnaissance survey and thirty minutes PP (Post Processing) static rover shoots observation using ProMark 3 Differential Global Positioning System (DGPS) was carried out on all the established points with instrument on station MCP03 as shown in the Figure 3 below:

3.2 Data Processing

Data processing follows some basic mathematical principles even though all were carried out using software. Raw DGPS data was processed using GNSS solutions and the results obtained were imputed into geoid height calculator software and the geoidal undulations were obtained with reference to EGM96.

3.2.1 Processing DGPS raw data using GNSS solutions

GNSS solution is an indispensable software tool for all surveyors who need to be efficiently and smoothly assisted in their surveys. GNSS Solutions really offers high standards of performance, processing speed, compactness and flexibility. It is extremely user-friendly, simplifying many of the office tasks, a feature which will be appreciated by novice and experienced users alike. GNSS Solutions support a wide range of surveying applications, whether conducted in post-processing or real time. This work was carried out in post processing format. Raw data collected by a receiver was processed to determine the differential relationship between the points occupied during data collection. The result of processing GPS raw data is a vector defining this relationship. Computation of these vectors is the role of the data processing module within GNSS Solutions. The data processing module automatically analyzes the quality of the raw data files and adjusts processing parameters to produce the best vector possible, transferring most of the processing effort from the user to the processing software. The output results after the processing and adjustment is the coordinates of the surveyed points with their associated variances and error ellipses as shown in Fig. 4 and 5.
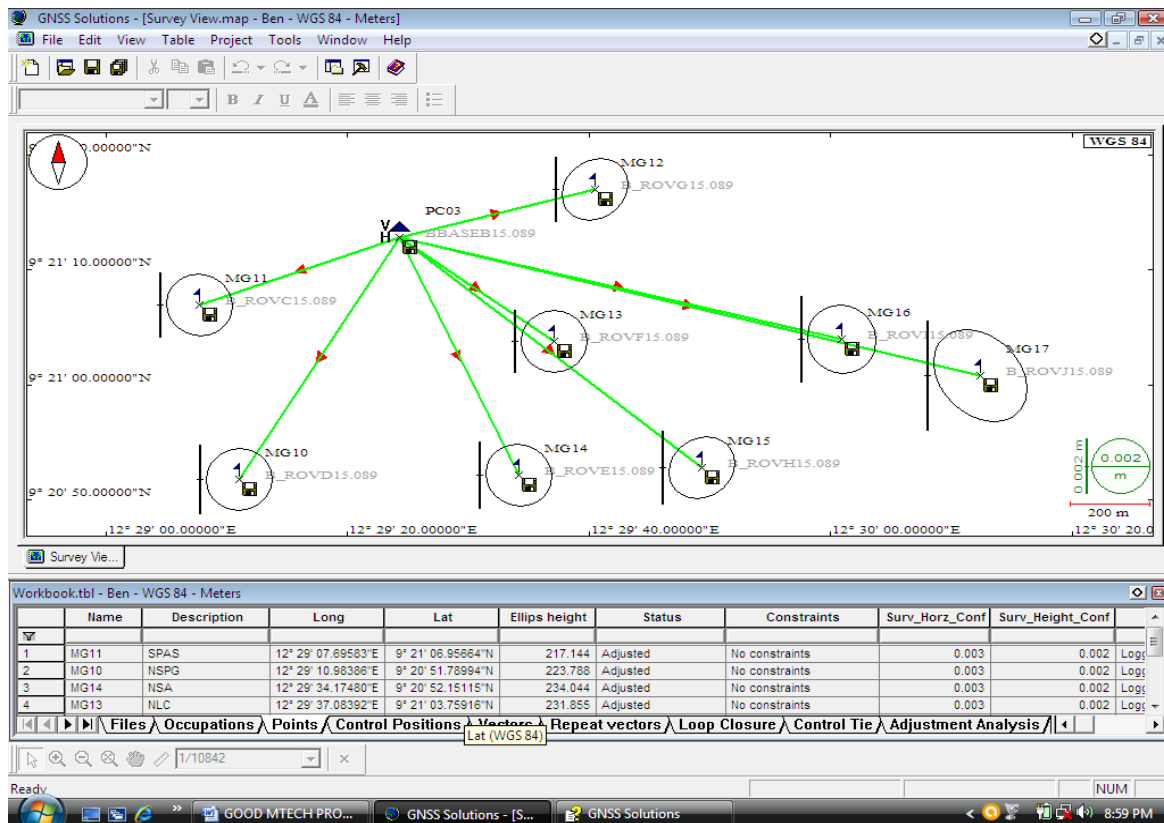

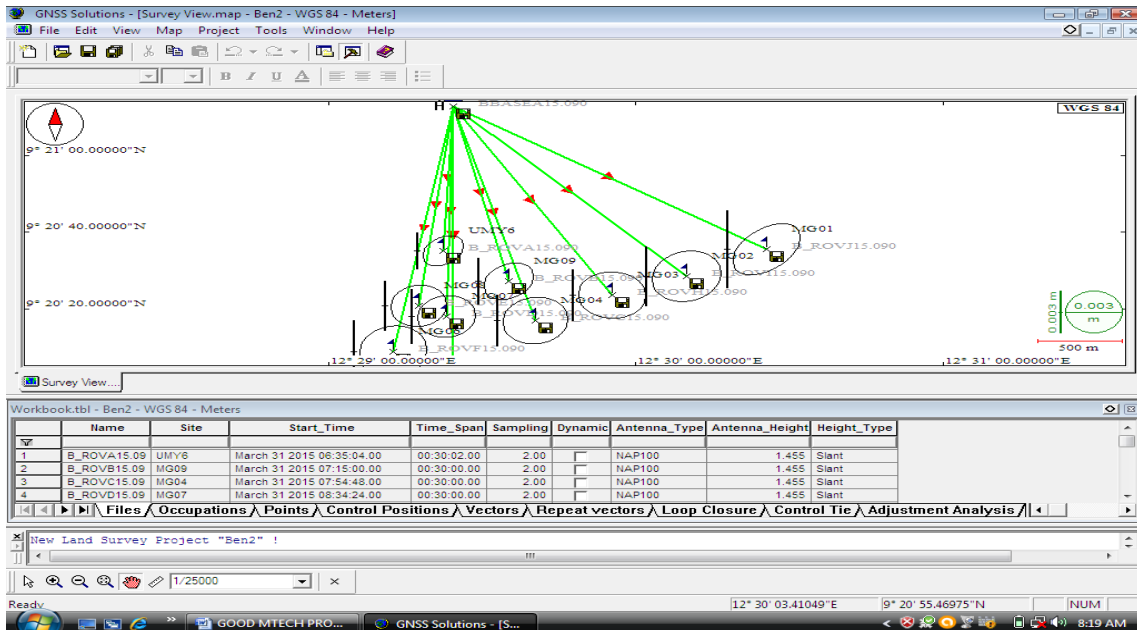**Figure 4:** Processing first session of observations.

Figure 5: Processing second session of observations.

3.2.2 Determination of geoidal undulation with reference to various EGMs

The Geoid Height Calculator calculates a geoid undulation at a point whose latitude and longitude is specified. The program is designed to use the potential coefficient model and a set of spherical harmonic coefficients of a correction term. The correction term is composed of several different components, the primary one being the conversion of a height anomaly to a geoid undulation. The principles of this procedure were initially described by Rapp (1996). The undulation will refer to the WGS84 ellipsoid.

When the disturbing potential T, at a point P(ϕ, λ, r) on or above the earth's surface, the height anomaly ζ is then given as equation (2), (Heiskanen and Moritz, 1967).

$$\zeta(\phi, \lambda, r) = \frac{T(\phi, \lambda, r)}{\Upsilon_B} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

Where r is the geocentric distance to the point, Υ_B is the value of theoretical gravity at point P. The conversion of height anomaly to a geoid undulation is given equation (3) as:

$$N(\phi, \lambda) = \zeta(\phi, \lambda, r) + \frac{\Delta g_B H}{\Upsilon} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (3)$$

Where Δg_B is the Bouguer anomaly at point P, H is the orthometric height at point P, and Υ is the average value of gravity at that point. The orthometric height can in turn be computed from equation (1). The output of computed undulations and orthometric heights with respect to the different EGMs used (EGM84 Geoid Height Calculator gives results to three decimal places, EGM96 Geoid Calculator gives results to two decimal places and GeoidEval (EGM2008) Calculator gives results to four decimal places) were presented in next section.

Different sets of geoidal undulation values computed from different Earth Gravity Models were plotted using Surfer 11 software. The 'Map' file was selected and the plotting commands were displayed. The wireframe and contour map commands were selected one after the other and the respective maps were automatically displayed.

3.3 Statistical Testing
3.3.1 Standard error and standard deviation

The standard error and standard deviation of the undulation values for each set of values was determined using statistical test calculator (STATCAL). In this, mean, unbiased standard deviation, standard error (variance) of the mean, and median were calculated. Data is entered using a standard spreadsheet interface (see Fig. 6). Finite population correction is incorporated into the calculation of the standard error of the mean, so the population size should be specified whenever the sample size is greater than ten percent of the population size. The software works on the principle that let the sample be x and the sample size be n. The Mean and Standard Deviation are given by equation (4) and (5) respectively.

$$\bar{x} = \frac{\sum x}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$S = \sqrt{\frac{\sum (\bar{x} - x)}{n-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

While Variance, $\sigma = S^2$. Results are shown in Figure 6 and Table 3.
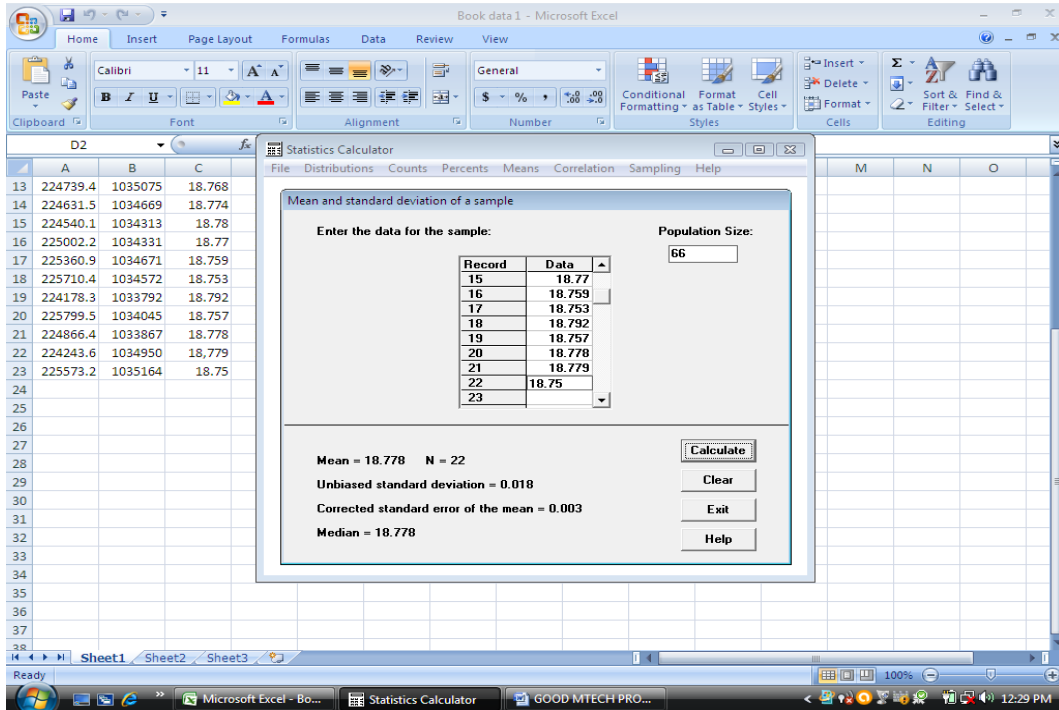

Figure 6: Window for computing standard deviation and variance of various undulations.

3.3.2 Chi square probability test

The chi-square is one of the most popular statistics because it is easy to calculate and interpret. Using the chi-square statistic and its associated degrees of freedom, the software reports the probability that the differences between the observed and expected frequencies occurred by chance. Generally, a probability of 0.05 or less is considered to be a significant difference. The result of this test is 0.148 which shows that the difference is not significant in the geoidal undulation values determined with respect to all earth gravity models.

IV. Presentation and Discussions of Results

4.1 Presentation of Results

The results presented are the DGPS derived adjusted data (Longitude, Latitude and ellipsoidal height) of all the stations selected in the study area (Table 1), the geoidal undulation values, the ellipsoidal heights and their respective orthometric heights computed from adjusted GPS data with reference to different Earth Gravity Models (EGMs) in meters (Table 2). Table 3 shows the Standard Deviation (SD) and Variance computed from each set of geoidal undulation values determined with respect to each gravity model as well as the result from chi square probability test. Table 4 shows the geographical and Universal Transverse Mercator (UTM) coordinates of all the selected stations in the study area.

Fig. 7(a), 8(a) and 9(a) shows the wireframes of the geoidal undulation values with reference to different earth gravity models while Fig. 7(b), 8(b) and 9(b) shows the contour maps plotted from the geoidal undulation values of EGM84, EGM96 and EGM2008 models respectively. All the results presented are the true reflection of the computational processes involved in the process of geoidal undulation determination using GPS observations. Figures 7(a) to 9(b) were all plotted from the longitude, latitude and the geoidal undulation values obtained.

Table 1: Adjusted Data of DGPS Observation.

Station	Station description	Longitude	Latitude	Ellipsoidal heigh	Status	Baseline length
MG11	SPAS	12° 29' 07.69"E	9° 21' 06.95"N	217.14	Adjusted	538.656m
MG10	NSPG	12° 29' 10.98"E	9° 20' 51.78"N	223.78	Adjusted	764.260m
MG14	NSA	12° 29' 34.17"E	9° 20' 52.15"N	234.04	Adjusted	703.359m
MG13	NLC	12° 29' 37.08"E	9° 21' 03.75"N	231.85	Adjusted	479.208m
MG12	NEC	12° 29' 40.52"E	9° 21' 17.01"N	225.01	Adjusted	510.650m
MG15	SMIT	12° 29' 49.30"E	9° 20' 52.85"N	240.72	Adjusted	978.972m
MG16	KUH	12° 30' 00.96"E	9° 21' 03.99"N	236.99	Adjusted	1150.871m
MG17	TED	12° 30' 12.44"E	9° 21' 00.86"N	241.00	Adjusted	1513.992m
MCP03	NTEB	12° 29' 24.32"E	9° 21' 12.84"N	221.56	Adjusted	Base station
UMY6	BH	12° 29' 22.43"E	9° 20' 35.14"N	223.21	Adjusted	1159.318m
MG09	BCA	12° 29' 35.14"E	9° 20' 27.08"N	229.92	Adjusted	1444.012m
MG04	PQTS	12° 29' 40.48"E	9° 20' 17.00"N	237.32	Adjusted	1784.965m
MG07	CPC	12° 29' 23.00"E	9° 20' 17.92"N	232.88	Adjusted	1687.462m
MG08	NNM	12° 29' 17.60"E	9° 20' 20.74"N	229.13	Adjusted	1613.210m
MG06	NBM	12° 29' 12.65"E	9° 20' 08.80"N	234.00	Adjusted	1998.926m
MG05	BMC	12° 29' 24.30"E	9° 19' 50.04"N	248.26	Adjusted	2543.491m
MG03	TTC	12° 29' 55.38"E	9° 20' 23.62"N	236.83	Adjusted	1784.782m
MG02	BCSQ	12° 30' 09.96"E	9° 20' 28.48"N	244.70	Adjusted	1948.991m
MG01	MG	12° 30' 25.56"E	9° 20' 35.64"N	248.26	Adjusted	2190.861m
MCP01	FSP	12° 30' 15.47"E	9° 20' 43.72"N	248.53	Adjusted	1800.189m
MCP02	CHC	12° 29' 44.96"E	9° 20' 37.73"N	232.10	Adjusted	1249.575m
MCP04	NPG	12° 30' 07.81"E	9° 21' 20.09"N	233.88	Adjusted	1346.743m

Source: Extracted from DGPS processed data (2015).

Table 2: Ellipsoidal heights, Geoidal Undulations and Orthometric Heights of Stations.

Station	h (m)	EGM84		EGM96		EGM2008	
		N (m)	H (m)	N (m)	H (m)	N (m)	H (m)
MG01	248.262	18.753	229.509	16.780	231.482	16.399	231.863
MG02	244.700	18.765	225.935	16.788	227.912	16.399	228.301
MG03	236.832	18.776	218.056	16.796	220.036	16.399	220.433
MG04	237.326	18.787	218.539	16.804	220.522	16.400	220.925
MG05	248.261	18.806	229.455	16.821	231.440	16.408	231.853
MG06	234.000	18.807	215.193	16.817	217.183	16.402	217.598
MG07	232.884	18.798	214.086	16.810	216.074	16.400	216.485
MG08	229.132	18.800	210.332	16.811	212.321	16.398	212.734
MG09	229.920	18.787	211.133	16.802	213.118	16.397	213.523
MG10	223.788	18.794	204.994	16.801	206.987	16.389	207.399
MG11	217.144	18.791	198.353	16.796	200.348	16.384	200.760
MG12	225.010	18.768	206.243	16.781	208.229	16.384	208.627
MG13	231.855	18.774	213.081	16.787	215.068	16.387	215.468
MG14	234.044	18.780	215.264	16.792	217.252	16.390	217.654
MG15	240.721	18.770	221.951	16.787	223.934	16.390	224.331
MG16	236.994	18.759	218.235	16.778	220.216	16.388	220.607
MG17	241.001	18.753	222.248	16.775	224.226	16.390	224.611
UMY6	223.219	18.792	204.427	16.803	206.416	16.394	206.825
MCP01	248.531	18.757	229.774	16.781	231.750	16.395	232.137
MCP02	232.102	18.778	213.324	16.794	215.308	16.395	215.707
MCP03	221.564	18.779	202.785	16.788	204.776	16.384	205.181
MCP04	233.884	18.750	215.134	16.770	217.114	16.385	217.499

Source: Extracted from the results of geoidal undulation computations (2015).

Table 3: Standard Deviation and Variance of Various Sets of Undulation Values.

	Standard deviation	Variance
EGM84 Geoidal undulation values	0.018	0.0003
EGM96 Geoidal undulation values	0.014	0.0002
EGM2008 Geoidal undulation values	0.007	0.0001
Chi Square probability test	0.05 or less CV	0.148 Obtained

Source: Compiled from statistical tests results (2015).

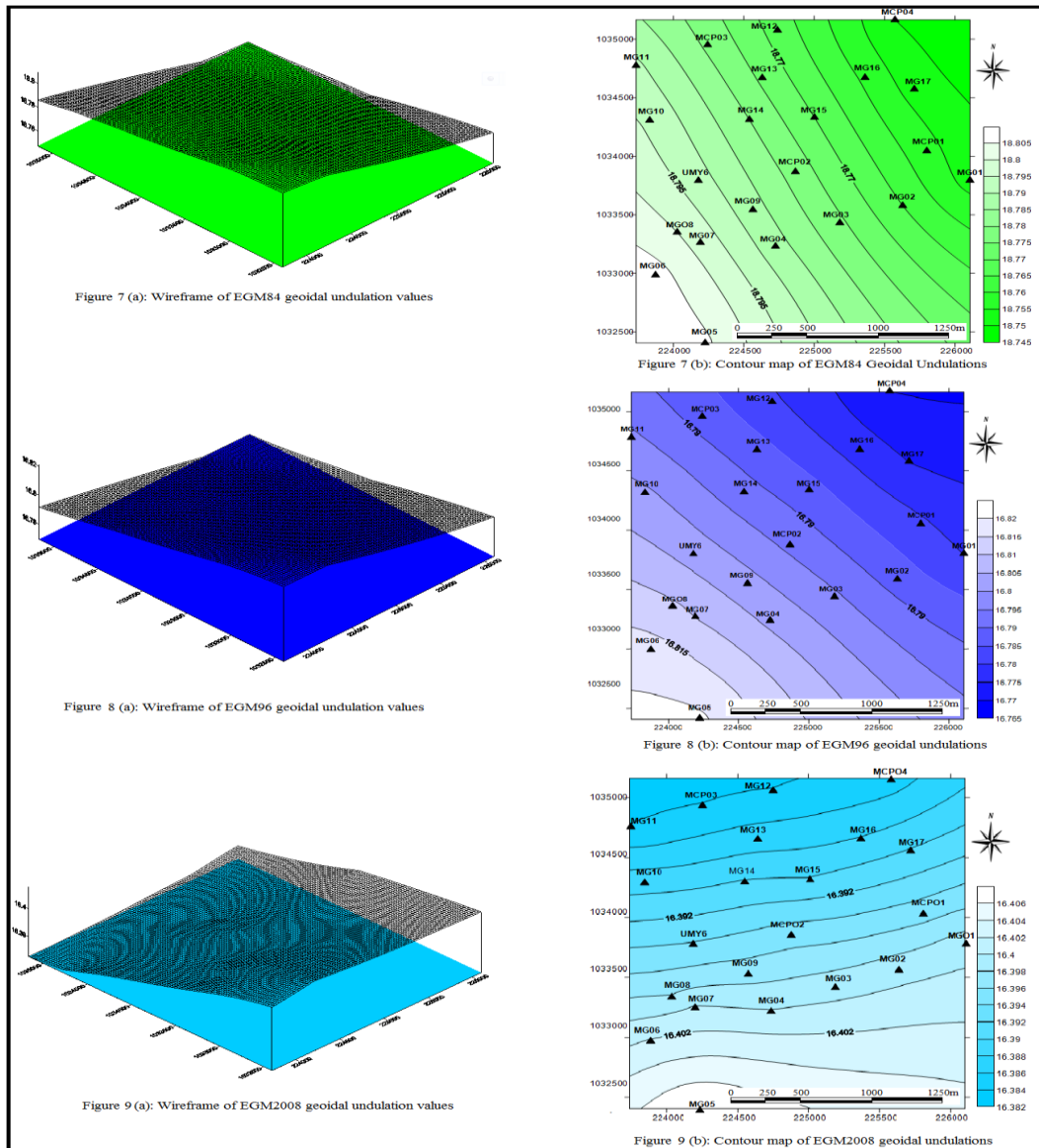**Table 4:** Geographical and UTM Coordinates of the Stations.

Station	Longitude (λ)	Latitude (φ)	E (m)	N (m)
MG01	12° 30' 25.56"E	9° 20' 35.64"N	226105.47	1033794.42
MG02	12° 30' 09.96"E	9° 20' 28.48"N	225627.94	1033577.41
MG03	12° 29' 55.38"E	9° 20' 23.62"N	225181.75	1033431.19
MG04	12° 29' 40.48"E	9° 20' 17.00"N	224725.39	1033230.94
MG05	12° 29' 24.30"E	9° 19' 50.04"N	224225.51	1032406.11

MG06	12° 29' 12.65"E	9° 20' 08.80"N	223873.61	1032985.26
MG07	12° 29' 23.00"E	9° 20' 17.92"N	224191.91	1033263.01
MG08	12° 29' 17.60"E	9° 20' 20.74"N	224027.35	1033351.17
MG09	12° 29' 35.14"E	9° 20' 27.08"N	224564.55	1033541.92
MG10	12° 29' 10.98"E	9° 20' 51.78"N	223832.03	1034306.67
MG11	12° 29' 07.69"E	9° 21' 06.95"N	223735.22	1034773.65
MG12	12° 29' 40.52"E	9° 21' 17.01"N	224739.41	1035075.42
MG13	12° 29' 37.08"E	9° 21' 03.75"N	224631.49	1034668.91
MG14	12° 29' 34.17"E	9° 20' 52.15"N	224540.11	1034312.69
MG15	12° 29' 49.30"E	9° 20' 52.85"N	225002.19	1034331.23
MG16	12° 30' 00.96"E	9° 21' 03.99"N	225360.91	1034671.10
MG17	12° 30' 12.44"E	9° 21' 00.86"N	225710.40	1034572.42
UMY6	12° 29' 22.43"E	9° 20' 35.14"N	224178.27	1033792.42
MCP01	12° 30' 15.47"E	9° 20' 43.72"N	225799.48	1034044.95
MCP02	12° 29' 44.96"E	9° 20' 37.73"N	224866.39	1033867.14
MCP03	12° 29' 24.32"E	9° 21' 12.84"N	224243.61	1034950.46
MCP04	12° 30' 07.81"E	9° 21' 20.09"N	225573.24	1035164.47

Source: Compiled from results of conversion from geographical to UTM coordinates (2015).

4.2 Discussion of Results

The geoidal undulation values in the study area are all positive and the implication is that the geoid at all points is above the ellipsoid. This result is seen to be the consequence of two possible factors: The geoid is affected by the distribution of mass of land above Mean Sea Level and the elevated area whose centre of gravity is outside the ellipsoid causes an upward attraction leading to local elevation of the geoid above the ellipsoid.

Figure 7 (a): Wireframe of EGM84 geoidal undulation values
Figure 7 (b): Contour map of EGM84 Geoidal Undulations
Figure 8 (a): Wireframe of EGM96 geoidal undulation values
Figure 8 (b): Contour map of EGM96 geoidal undulations
Figure 9 (a): Wireframe of EGM2008 geoidal undulation values
Figure 9 (b): Contour map of EGM2008 geoidal undulations

The other factor is that the mass excess under the ellipsoid forces equipotential surface (geoid) to warp upward which gives positive geoid undulation over mass excess under the ellipsoid. The geoid undulation in the study area is affected by relatively flat topography and the heterogeneous mass distribution of the earth surface due to developmental changes in the school since its inception in 1981. As structural development increases in the school, the mass distribution changes and consequently the gravity values at different points.

The geoidal undulation value at a point which is a function of gravity at that point also changes as a function of time. This can be observed from Fig. 7(a), 8(a) and 9(a) where wireframe plotted from geoidal undulation values of different earth gravity models of different degrees and order of accuracy (EGM84, EGM96 and EGM2008) shows significant changes with respect to time as well as differences in accuracy between the EGMs due to changes in gravity. EGM96 is better than EGM84 and EGM2008 is better than EGM96. The contour maps (Fig. 7(b), 8(b) and 9(b)) of these undulations values across EGM documentations show the changes in slope directions as the shape of the geoid changes.

EGM2008 referred to the WGS84 has been found to be the most suitable gravity model for computation of geoidal undulation values of selected stations due to the fact that additional spherical harmonic coefficients has been added extending to degree 2190 and order 2159. This made the model suitable for the study of long or short wavelength of the geoidal undulation values. This claim can be verified from Table 3 where the variance of EGM 2008 set of geoidal undulation values (0.001) is the smallest among the three variances. Also from Table 2, the average difference between EGM84 and EGM96 geoidal undulation is 2 metres but the difference between EGM96 and EGM2008 is 0.4m across 12 years difference each. These changes attest to the fact that geoid changes as the mass of land above Mean Sea Level experience changes as a result of natural changes and man's activity. The geoid undulation shows decrease in the study area from average of 18.7784m to 16.7937m in 12years period and further decreased from 16.7937m to 16.3935m over the same period.

V. Conclusion and Recommendations

From the result obtained from this study, it was observed that the local geoidal undulation values in Modibbo Adama University of Technology Yola are all positive in conformity with the previous works carried out which states that "the geoid undulations determined for Nigeria are all positive". Significant changes were observed between the undulation values with respect to EGM84 and EGM2008. The average difference between EGM84 and EGM96 undulation values is about 2m across the area of survey while the average difference between EGM96 and EGM2008 is 0.4m, owing to the refinement of the earth gravity model to higher degree and order of accuracy. It was also observed that geodesy has evolved from determining three dimensional parameters to what is known as "four dimensional geodesy". This is due to the daily changes in its parameters resulting from physical changes experienced on the earth surface. Time therefore has become an important Parameter in geodesy where it seeks to determine longitude (λ), latitude (ϕ), height (h) and time (t) to define the figure of the earth. Geoid therefore is undulating in form and its shape changes as time changes.

Geoidal undulation and Orthometric height determination using Geoid Calculator Software packages is therefore recommended over expensive and time consuming conventional method of geodetic leveling and gravity measurement due to its simplified approach to data collection, processing and analysis.

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